7. S. V. Tarakanov, I. Yu. Pavlov, O. M. Todes, and A. D. Gol'tsiker, "Analysis of formation of a relaxation shock wave in an air suspension of chemically inert liquid particles," Vzryvn. Del., No. 80/37 (1978).
8. T. R. Amanbaev and A. I. Ivandaev, "Shock wave structure in two-phase mixtures of gas with liquid drops," Prikl. Mekh. Tekh. Fiz., No. 2 (1988).
9. O. G. Engel, "Fragmentation of water drops in the zone behind an air shock," J. Res. Nat. Bur. Stand., 60, No. 3 (1958).
10. A. A. Ranger and J. A. Nicholls, "Aerodynamic scattering of liquid drops," AIAA J., 7, No. 2 (1969).
11. A. A. Borisov, B. E. Gel'fand, M. S. Natanzon, and O. M. Kossov, "Drop fragmentation regimes and criteria for their existence," Inzh. Fiz. Zh., 40, No. 1 (1981).
12. I. P. Bazarov, Thermodynamics [in Russian], Vysshaya Shkola, Moscow (1983).
13. O. M. Belotserkovskii and Yu. M. Davydov, The Coarse Particle Method in Gas Dynamics [in Russian], Nauka, Moscow (1982).
14. A. A. Gubaidullin, A. I. Ivandaev, and R. I. Nigmatulin, "A modified 'coarse particle' method for calculation of nonsteady wave processes in multiphase dispersed media," Zh. Vychisl. Mat. Mat. Fiz., 17, No. 6 (1977).
15. A. I. Ivandaev and A. G. Kutushev, "Numerical study of nonsteady wave flows of gas suspensions with detection of boundaries of two-phase regions and contact discontinuities in the carrier gas," ChMMSS, 14, No. 6 (1983).

CHANGE IN THE SHAPE AND CHARACTERISTICS OF A BURNING BODY IN HYPERSONIC FLOW

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The investigation of hypersonic flow around burning models is of interest in order to develop a mathematical theory of internal ballistics, to understand the combustion of solid fuel as it exists from an engine nozzle, to simulate the physical processes in meteoric phenomena [1], and to study features of the combustion and detonation of explosive gas mixtures. By now, stationary combustion of carbon or materials with carbon thermal protective coatings have been rather completely studied [2]. At the same time, no such analysis exists for material with a complex arbitrary chemical composition, and it is necessary to use mainly empirical data. One problem which has not been investigated is how the surface of a body burns in hypersonic motion and how its aerodynamic characteristics change. Results from a ballistic range of experiments have been presented [1] on mass removal from rapidly burning models of made of pyrotechnic materials and on the hypersonic flow around them.

Here we find the shape change when spherical or parabolic bodies are burned, and we also find the resistance and mass after the hypersonic motion on a ballistic track under the same conditions as in [1]. We calculated how the radius of curvature and the lateral area, which determines the luminosity of the burning models, changes with time.

1. Basic Concepts and Assumptions. Today there are a large number of different mechanisms which explain the combustion of solid fuels of a given composition [3]. A simplified combustion model for the solid surface of a pyrotechnic powder is as follows. It is assumed that the chemical reaction is initiated by instantaneous ignition of the model in a barrel [4] and then proceeds by a very simple method: oxygenated fuel → gaseous reaction products. All heat going from the reaction zone to the solid phase is sufficient to maintain continuous combustion of the thermal flow. It is assumed [3, 4] that the temperature of the burning surface is constant, that the combustion is one-dimensional and goes layer by layer, and that the material is gasified in a narrow zone at the surface. The gas phase is treated as a quasi-stationary phase which instantaneously adds to the thermal state of the surface layer.

According to current ideas, the flow of combustion products which move along the surface has a strong effect on the heat and mass transfer. Turbulization of the boundary layer intensifies the transfer processes and also increases convective heat transfer, which increases

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the surface combustion velocity. This phenomenon is called erosive combustion [3, 4]. In the standard theories of erosive combustion, there is no way to validate the use of relationships from the theory of a turbulent non-reacting boundary layer to describe the transfer processes in the combustion zone [3]. If the basic characteristics of all the physicochemical processes were well known, then it would be possible to calculate the linear combustion velocity of the surface as a function of the pressure, temperature, and composition, i.e., the velocity of the phase-separation boundary relative to an immobile solid phase. However, processes which accompany erosive combustion have not been completely investigated to date [3-5]. Therefore an extrapolation formula, which gives the combustion velocity in the form [3, 4]

$$v_n = A p_w^{\nu} \tag{1.1}$$

is widely used to reduce experimental data on the combustion of powders. Here A and ν are empirical constants and p_w is the pressure on the surface of the body.

Experimental data [1] on the radiation from (combustion) gases have been reduced [6] to obtain the constants A and ν which are used below. The combustion velocity (1.1) is limited by the reaction rate in the gas phase [4], which provides the thermal flow required to maintain continuous combustion. This leads to a constant surface temperature T_w equal to the material decomposition temperature [4]. For powders, the transformation of the solid material into a gas occurs irreversibly due to the chemical decomposition reaction. The assumption of a constant surface temperature of the burning material is correct if this reaction has large activation energies [3]. Combustion theory in the constant-temperature approximation is called the theory of ideal combustion [4]. The combustion process is ideal in the sense that a major assumption is made about the decomposition kinetics.

Evaluations of dimensionless parameters [1, 6] have shown that gaseous products are injected rapidly enough (into the combustion zone) from the surface of this material to use the asymptotic strong-injection model [7]. In this case the flow in the shock layer can be represented by three subregions: nonviscous air flow behind the shock, a nonviscous injection region near the surface, and a thin "floating" boundary layer between them. The asymptotic solution to the gas dynamic equations in the injection layer and a comparison with numerical calculations [6, 7] have shown that the pressure distribution near the surface follows Newton's formula with a high degree of accuracy. The flow structure can change when there is heat evolution near the surface; however, hereafter it is assumed that the pressure distribution does not change it (or changes it only through the change in shape of the burning surface).

2. Solution to the Equation for the Shape Change of the Burning Body. Now we examine the problem of mass erosion and the change in shape of a three-dimensional body which burns at the surface and moves at a hypersonic velocity. We only consider motion at a zero attack angle, with no lift or rotation.

Here the surface of the body is treated as a shear surface, whose shape and motion must be found by the solution process. Let x, y, and z be rectangular coordinates, where the z-axis points into the oncoming flow. If the equation for the surface is given explicitly z = z(x, y, t), then the displacement velocity along the surface normal is given by [8]

$$v_n = \frac{-\dfrac{\partial z}{\partial t}}{\left[1+\left(\dfrac{\partial z}{\partial x}\right)^2+\left(\dfrac{\partial z}{\partial y}\right)^2\right]^{1/2}}. \tag{2.1}$$

Equation (2.1) describes the change in shape of a body when v_n is a known function of the body geometry, the physical surface processes, and an initial condition $z = z_0(x, y, 0)$ at t = 0. The combustion velocity in the form of (1.1) is an additional condition.

At hypersonic velocities, the pressure distribution for a wide class of bodies in uniform flow are well described by Newton's formula [6, 7]

$$p_w = \rho_\infty v_\infty^2(t)\, n_z^2; \tag{2.2}$$

$$n_z = \frac{1}{\sqrt{1+z_x'^2+z_y'^2}}; \tag{2.3}$$

$$z_x' = \frac{\partial z}{\partial x},\quad z_y' = \frac{\partial z}{\partial y}, \tag{2.4}$$

where $v_\infty(t)$ is the velocity of the body, and ρ_∞ is the density of the incoming gas flow. By substituting (1.1), the expression for the combustion velocity, and (2.3) into Eq. (2.1), we obtain

$$-\frac{\partial z}{\partial \tau} = (1 + p^2 + q^2)^{1/2-\nu} = H(p, q), \ \tau = \int_0^t A\left(\rho_\infty v_\infty^2(t)\right)^\nu dt, \ p = \frac{\partial z}{\partial x}, \ q = \frac{\partial z}{\partial y}. \tag{2.5}$$

Now we examine the solution to Eq. (2.5) in the case where the initial shape is given parametrically:

$$t = 0: \ x = s_1, \ y = s_2, \ z = z_0(s_1, s_2), \ p_0 = \frac{\partial z_0}{\partial s_1}, \ q_0 = \frac{\partial z_0}{\partial s_2}.$$

Then, by using the method of characteristics (Cauchy's method) [9], we find an explicit solution

$$\begin{gathered} x(\tau) = s_1 + \tau \frac{(1-2\nu) p_0}{\left(1 + q_0^2 + p_0^2\right)^{\nu+1/2}}, \ y(\tau) = s_2 + \tau \frac{(1-2\nu) q_0}{\left(1 + q_0^2 + p_0^2\right)^{\nu+1/2}}, \\ z(\tau) = z_0(s_1, s_2) - \tau \frac{2\nu\left(p_0^2 + q_0^2\right) + 1}{\left(1 + q_0^2 + p_0^2\right)^{\nu+1/2}}. \end{gathered} \tag{2.6}$$

Equation (2.6) describes the time-dependent shape change of a three-dimensional body. Now we examine more closely the case where the body is axisymmetric, with a surface equation $y = y(x, t)$ relative to the initial body shape $y = y_0(x)$, where x and y are Cartesian coordinates with the origin at the critical point and the Ox-axis pointed into the incoming flow and the Oy-axis normal to Ox. Then by the same reasoning we obtain the equation for the time-dependent shape change of an axisymmetric body:

$$-\frac{\partial y}{\partial \tau} = q^{2\nu}(1 + q^2)^{1/2-\nu} = H(q), \ \tau = \int_0^t A\left(\rho_\infty v_\infty^2\right)^\nu dt, \ q = y'_x. \tag{2.7}$$

If the initial conditions are specified as

$$t = 0, \ x = s, \ y = y_0(s), \ \frac{dy_0}{ds} = q_0,$$

then, by using the method of characteristics, we obtain the explicit solution

$$x(s, \tau) = s + \tau \frac{q_0^{2\nu-1}\left(2\nu + q_0^2\right)}{\left(1 + q_0^2\right)^{\nu+1/2}}, \ y(s, \tau) = y_0(s) + \tau \frac{q_0^{2\nu}(2\nu - 1)}{\left(1 + q_0^2\right)^{\nu+1/2}}. \tag{2.8}$$

We now compute the radius of curvature at any point on the body. Because the surface is specified by the equation $y = y(x, t)$, then

$$y(s, t) = y(x(s, t), t), \ y'_s = q_0 x'_s. \tag{2.9}$$

From (2.9) the radius of curvature is

$$R = \frac{1}{\left|\frac{dq_0}{ds}\right|}\left(1 + q_0^2\right)^{3/2}\left|1 + \tau \frac{(2\nu - 1) q_0^{2\nu-2}\left(2\nu - q_0^2\right)}{\left(1 + q_0^2\right)^{3/2+\nu}} \frac{dq_0}{ds}\right|,$$

from which we obtain that at $t = 0$

$$R_0 = \frac{1}{\left|\frac{dq_0}{ds}\right|}\left(1 + q_0^2\right)^{3/2}.$$

Finally the radius of the body takes the form

$$\frac{R}{R_0} = \left|1 + \tau \frac{(2\nu - 1) q_0^{2\nu-2}\left(2\nu - q_0^2\right)}{\left(1 + q_0^2\right)^{3/2+\nu}} \frac{dq_0}{ds}\right|. \tag{2.10}$$

The characteristics of Eq. (2.7) are straight lines. If we introduce φ and ψ – the inclination angles to the Ox axis of the initial generatrix of the body and an arbitrary characteristic – then, from (2.8), φ and ψ are related:

$$\operatorname{tg} \psi = \frac{\operatorname{tg} \varphi\,(2\nu - 1)}{2\nu + \operatorname{tg}^2 \varphi}.$$

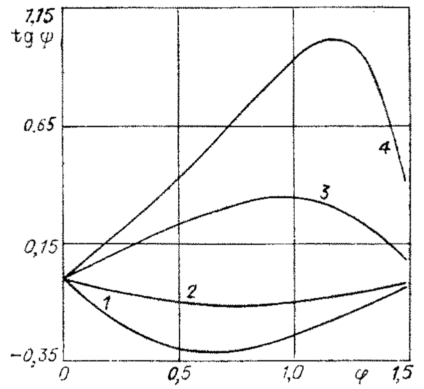

Fig. 1

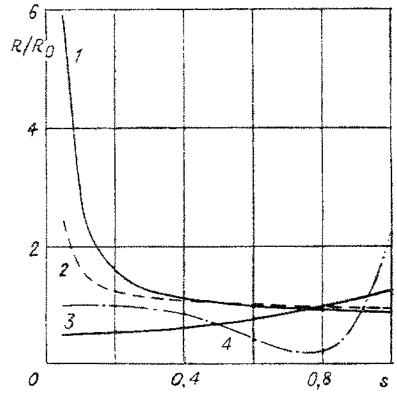

Fig. 2

Figure 1 shows $\tan\psi$ as a function of φ for $\nu = 0.27$, 0.4, 1.0, and 3.0 (curves 1-4, respectively). The function $\psi = f(\varphi)$ at $\tan\varphi = \sqrt{2\nu}$ has an extremum of $\tan\psi = \sqrt{2\nu}\,(2\nu - 1)/4\nu$. The extremum is a maximum for $\nu > 1/2$ and a minimum for $\nu < 1/2$.

When $\nu = 1/2$, the characteristics of Eq. (2.7) are parallel to the Ox-axis. Later it will be shown that the case $\nu = 1/2$ is the critical one for blunt bodies.

3. Change in the Shape, the Mass, and the Resistance Coefficient for a Spherical Body. All the relationships obtained above are valid for any axisymmetric body. Now we examine the particular case where the initial shape is a sphere of initial radius R_0. Then its shape is written as

$$x_0(s) = R_0(1 - s),\; y_0(s) = R_0\sqrt{1 - s^2},\; q_0 = \frac{dy_0}{ds} = \frac{s}{\sqrt{1 - s^2}}. \tag{3.1}$$

By substituting it into (2.8) we obtain

$$\begin{aligned} x &= R_0(1 - s) + \tau s^{2\nu-1}[2\nu + (1 - 2\nu)s^2], \\ y &= R_0\sqrt{1 - s^2} + \tau(2\nu - 1)s^{2\nu}\sqrt{1 - s^2}. \end{aligned} \tag{3.2}$$

Equation (3.2) describes the time-dependent shape change of the body. Now we find an expression for the radius of curvature of the body at any point. To do this we substitute Eq. (3.1) into Eq. (2.10):

$$R/R_0 = 1 - \sigma[2\nu(2\nu - 1)s^{2\nu-2} + (1 - 4\nu^2)s^{2\nu}], \quad \sigma = \tau/R_0.$$

At the critical point ($s = 1$) it then follows that

$$R/R_0 = 1 + (2\nu - 1)\sigma.$$

Figure 2 shows R/R_0 as a function of s for $\sigma = 0.2$ and various ν's. Curves 1-4 corresponds to $\nu = 0.27$, 0.4, 1.0, and 3.0. We note that the change in the radius R(s) is nonmonotonic for $\nu > 1$.

It can be seen that when $\nu > 1/2$, the body at the critical point becomes more blunt as time progresses. When $\nu = 1/2$, the radius of curvature remains constant, not only at the critical point, but also at all others. In this case the body "burns" parallel to itself. For a blunt nose we obtain from (3.2) that, as $s \to 1$ and $q_0 \to \infty$, the time-dependent change in its coordinates is $x(0, \tau) = \tau$. When $\nu > 1/2$, the point $s = 0$ remains invariant with time in our coordinate system. If $\nu < 1/2$, the position of the point $s = 0$ does change with time.

Now we present relationships for the time-dependent change of the head drag coefficient C_D, which has the form

$$C_D = C_p + C_R,$$

where C_p is the wave-resistance coefficient:

$$C_p = R^2\left[\frac{1}{2}\rho_\infty v_\infty^2 F\right]^{-1}\int_0^{2\pi}\int_0^{\theta_*} p_w(\theta)\cos\theta\sin\theta\, d\theta\, d\varphi;$$

C_R is the reactive-force coefficient, including injection:

$$C_R = R^2 \left[\frac{1}{2}\rho_\infty v_\infty^2 F\right]^{-1} \int_0^{2\pi}\int_0^{\theta_*} \rho_w(\theta)\, v_w^2(\theta) \cos\theta \sin\theta \, d\theta \, d\varphi;$$

F is the variable area of the maximum cross section; and ρ_W and v_W are the velocity and density of the injected gases.

An estimate gives the relationship

$$\frac{C_R}{C_p} = \delta_0^2 = \frac{\rho_w^* v_w^2}{\rho_\infty v_\infty^2} \sim 0{,}01 \ll 1.$$

Therefore, we shall assume that $C_D \approx C_p$. By using Newton's formula (2.2), we obtain

$$C_p = \frac{4\pi}{F}\int_\tau^R \frac{y y_x'^3 \, dx}{1 + y_x'^2}. \tag{3.3}$$

Now we examine the change of the body mass with time. By using the law of conservation of mass we write

$$\frac{dM}{dt} = -\int_{F_1} (\rho_1 v_n) \, dF_1 ,$$

where ρ_1 is the density of the body and dF_1 is an element of surface area of the body.

By using Eq. (1.1) and Newton's formula for the pressure on the body surface, and then going from differentiation with respect to t to differentiation with respect to τ, we find

$$\frac{dM}{d\tau} = -2\pi\rho_1 \int_\tau^{R_0} \frac{y y_x'^{2\nu}}{(1 + y_x'^2)^{\nu - 1/2}} dx.$$

The formula for the lateral surface area F_1 as a function of time has the form

$$F_1 = 2\pi \int_\tau^{R_0} y \sqrt{1 + y'^2} dx. \tag{3.4}$$

Now we examine three cases separately ($\nu < 1/2$, $\nu > 1/2$, and $\nu = 1/2$) and find the head drag coefficient and the mass as functions of time.

1. $\nu > 1/2$. By using Eqs. (3.1) and (3.2) and then changing the variable integration from x to s in Eq. (3.3), we obtain

$$C_p = 1 + \sigma \frac{8\nu - 4}{(\nu + 2)(\nu + 1)} + \sigma^2 \frac{(2\nu - 1)^2}{2(2\nu + 2)(2\nu + 1)}, \quad \sigma = \frac{\tau}{R_0}.$$

Here $F = \pi R_0^2$, because the area of the maximum midsection does not change with time; $C_p(0) = 1$ at $\tau = 0$.

Now we examine the change in the body mass with time. From Eqs. (3.1) and (3.2), we find

$$\frac{dM}{d\tau} = -2\pi\rho_1 \left[\frac{R_0^2}{2\nu + 1} + R_0\tau \frac{2(2\nu - 1)^2}{16\nu^2 - 1} + \tau^2 \frac{(2\nu - 1)^3}{36\nu^2 - 1}\right].$$

We integrate this expression over τ and divide by $M_0 = 2/3\pi R_0^3 \rho_1$; then

$$\frac{M}{M_0} = 1 - \frac{3\sigma}{2\nu + 1} - \frac{3(2\nu - 1)^2\sigma^2}{16\nu^2 - 1} - \frac{(2\nu - 1)^3\sigma^3}{36\nu^2 - 1}.$$

Thus, for $\nu > 1/2$ we obtain the required time-dependent functions for the head drag coefficient and the mass. By analogous reasoning, we have for the area of the lateral surface of the body from (3.4):

$$\frac{F_1}{F_{10}} = 1 - \frac{2\sigma}{2\nu + 1} - \frac{(2\nu - 1)^2}{16\nu^2 - 1}\sigma^2,$$

where F_{10} is the surface area at time t = 0.

2. $\nu = 1/2$. In this case the equations for the change of body shape take the form

$$x = R_0(1 - s) + \tau, \quad y = R_0\sqrt{1 - s^2}. \tag{3.5}$$

Then

$$C_p = (\pi R_0^2/F)(1-\sigma^4).$$

The time-dependent change in the radius of the maximum midsection must be known in order to find the final equation for C_p. It is obvious that the body has the largest vertical cross section at $x = R_0$; in this case the maximum midsection radius at this point is exactly y_0, which corresponds to $x = R_0$. Now we find this quantity. For $x = R_0$ and $s = \sigma$, it follows from Eq. (3.5) that

$$y = R_m = R_0\sqrt{1-\sigma^2},$$

where R_m is the radius of the maximum midsection. Then $F = \pi R_m^2 = \pi R_0^2(1-\sigma^2)$.

Finally, we obtain

$$\frac{C_p}{C_p(0)} = 1+\sigma^2, \quad \sigma = \frac{\tau}{R_0}.$$

Now we examine the mass loss from the body surface. By using Eq. (3.5), integrating, and then dividing by M_0, we finally have

$$\frac{M}{M_0} = 1+\frac{1}{2}\sigma^3-\frac{3}{2}\sigma.$$

Then we find the area of the lateral surface from Eq. (3.4):

$$F_1/F_{10} = 1-\sigma.$$

3. $\nu < 1/2$. We transform the integration variable in Eq. (3.3) from x to s using Eqs. (3.1) and (3.2). The point $x = \tau$ corresponds to the point $s = 1$, and the point $x = R_0$ corresponds to the point $s_0(\tau)$, which is the root of the equation

$$R_0 = R_0(1-s)+\tau s^{2\nu-1}[2\nu+(1-2\nu)s^2]. \tag{3.6}$$

Then, on the basis of (3.3), we obtain

$$\frac{C_p}{C_p(0)} = \frac{\pi R_0^2}{F}\Big[1-s_0^4(\tau)+\sigma(8\nu-4)\Big[\frac{\nu+1}{\nu+2}\left(1-s_0^{2\nu+4}(\tau)\right)-$$
$$-\frac{\nu}{\nu+1}\left(1-s_0^{2\nu+2}(\tau)\right)\Big]+2\sigma^2(2\nu-1)^2\Big[\frac{2\nu+1}{2\nu+2}\left(1-s_0^{4\nu-4}(\tau)\right)-\frac{2\nu}{2\nu+1}\left(1-s_0^{4\nu+2}(\tau)\right)\Big]\Big], \tag{3.7}$$

where F is the maximum midsection area, and R_0 is the body radius at time t = 0. We calculate the change in mass analogously:

$$\frac{dM}{d\tau} = -2\pi\rho_1\Big[\frac{R_0^2}{2\nu+1}\left(1-s_0^{2\nu+1}(\tau)\right)+R_0\tau(2\nu-1)\Big\{\frac{2\nu+2}{4\nu+1}\left(1-s_0^{4\nu+1}(\tau)\right)-$$
$$-\frac{2\nu}{4\nu-1}\left(1-s_0^{4\nu-1}(\tau)\right)\Big\}+\tau^2(2\nu-1)^2\Big\{\frac{2\nu+1}{6\nu+1}\left(1-s_0^{6\nu+1}(\tau)\right)-\frac{2\nu}{6\nu-1}\left(1-s_0^{6\nu-1}(\tau)\right)\Big\}\Big]. \tag{3.8}$$

We cannot find the explicit function $s_0(\tau)$ vs. τ required to integrate Eq. (3.8), so we attempt to find an approximation. To do this we examine Eq. (3.6) at $s = s_0$:

$$R_0 = \tau[2\nu s^{2\nu-2}+(1-2\nu)s^{2\nu}].$$

The second term on the right side of this equation is small compared to the first, so we neglect it:

$$R_0 = \tau 2\nu s^{2\nu-2}, \quad s_0(\tau) = (2\nu\sigma)^{1/(2-2\nu)}. \tag{3.9}$$

By substituting (3.9) into Eq. (3.7), we have

$$\frac{C_p}{C_p(0)} = \frac{\pi R_0^2}{F}\Big[1-(2\nu\sigma)^{2/(1-\nu)}+\sigma(8\nu-4)\Big\{\frac{\nu+1}{\nu+2}\left(1-(2\nu\sigma)^{(\nu+2)/(1-\nu)}\right)-$$
$$-\frac{\nu}{\nu+1}\left(1-(2\nu\sigma)^{(\nu+1)/(1-\nu)}\right)\Big\}+2\sigma^2(2\nu-1)^2\Big\{\frac{2\nu+1}{2\nu+2}\left(1-(2\nu\sigma)^{(2\nu+2)/(1-\nu)}\right)-\frac{2\nu}{2\nu+1}\left(1-(2\nu\sigma)^{(2\nu+1)/(1-\nu)}\right)\Big\}\Big]. \tag{3.10}$$

By replacing $s_0(\tau)$ in (3.8) by its value from (3.9) and integrating, we obtain

$$\frac{M}{M_0} = 1-3\Big[\frac{1}{2\nu+1}\left(\sigma-\frac{(2\nu)^{(2\nu+1)/(2-2\nu)}(2-2\nu)}{3}\sigma^{3/(2-2\nu)}\right)+$$
$$+(2\nu-1)\Big\{\frac{2\nu+2}{4\nu+1}\left(\frac{\sigma^2}{2}-\frac{(2\nu)^{(4\nu+1)/(2-2\nu)}(2-2\nu)}{5}\sigma^{5/(2-2\nu)}\right)-\frac{2\nu}{4\nu-1}\left(\frac{\sigma^2}{2}-\frac{(2\nu)^{(4\nu-1)/(2-2\nu)}(2-2\nu)}{3}\sigma^{3/(2-2\nu)}\right)\Big\}+$$

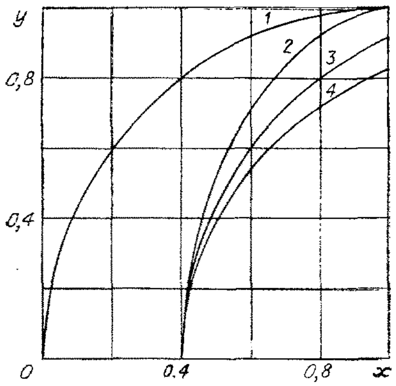

Fig. 3

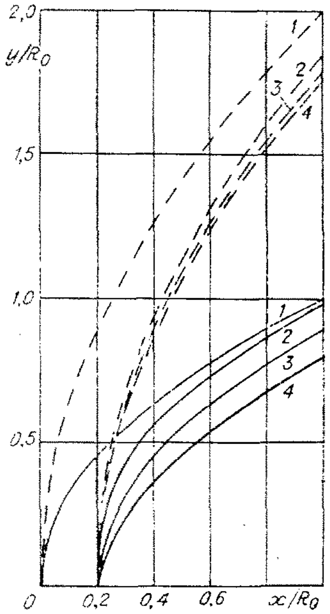

Fig. 4

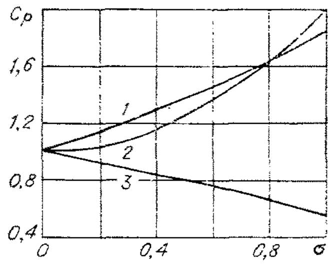

Fig. 5

$$+(2\nu-1)^2\left\{\frac{2\nu+1}{6\nu+1}\left(\frac{\sigma^3}{3}-\frac{(2\nu)^{(6\nu+1)/(2-2\nu)}(2-2\nu)}{7}\sigma^{7/(2-2\nu)}\right)-\frac{2\nu}{6\nu-1}\left(\frac{\sigma^3}{3}-\frac{(2\nu)^{(6\nu-1)/(2-2\nu)}(2-2\nu)}{5}\sigma^{5/(2-2\nu)}\right)\right\}\Bigg].$$

The time-dependent variation of the maximum-midsection area must be known in order to find the final time-dependent function for the head drag coefficient. It follows from Eq. (3.9) that

$$x=R_0,\quad s=(2\nu\sigma)^{1/(2-2\nu)},\quad y=(R_0+\tau(2\nu-1)s^{2\nu})\sqrt{1-s^2}.$$

Because the quantity $\tau(2\nu-1)s^{2\nu}$ is an order of magnitude smaller than R_0, we can write

$$y=R_m=R_0\sqrt{1-(2\nu\sigma)^{1/(1-\nu)}}.$$

Then

$$F=\pi R_m^2=\pi R_0^2\left(1-(2\nu\sigma)^{1/(1-\nu)}\right).$$

Finally we have from (3.10) that

$$\frac{C_p}{C_p(0)}=1+(2\nu\sigma)^{1/(1-\nu)}+\sigma(8\nu-4)\left\{\frac{\nu+1}{\nu+2}\frac{\left(1-(2\nu\sigma)^{(\nu+2)/(1-\nu)}\right)}{\left(1-(2\nu\sigma)^{1/(1-\nu)}\right)}-\right.$$

$$\left.-\frac{\nu}{\nu+1}\frac{\left(1-(2\nu\sigma)^{(\nu+1)/(1-\nu)}\right)}{\left(1-(2\nu\sigma)^{1/(1-\nu)}\right)}\right\}+2\sigma^2(2\nu-1)^2\left\{\frac{2\nu+1}{2\nu+2}\frac{\left(1-(2\nu\sigma)^{(2\nu+2)/(1-\nu)}\right)}{\left(1-(2\nu\sigma)^{1/(1-\nu)}\right)}-\frac{2\nu}{2\nu+1}\frac{\left(1-(2\nu\sigma)^{(2\nu+1)/(1-\nu)}\right)}{\left(1-(2\nu\sigma)^{1/(1-\nu)}\right)}\right\}.$$

By analogous reasoning, we obtain

$$\frac{F_1}{F_{10}}=-(2\nu\sigma)^{1/(2-2\nu)}+\sigma\left[2\nu\left((2\nu\sigma)^{(2\nu-1)/(2-2\nu)}-1\right)-\frac{(2\nu-1)(2\nu+2)}{2\nu+1}\left((2\nu\sigma)^{(2\nu+1)/(2-2\nu)}-1\right)\right]+$$

$$+\sigma^2(2\nu-1)^2\left[\frac{2\nu}{4\nu-1}\left((2\nu\sigma)^{(4\nu-1)/(2-2\nu)}-1\right)-\frac{2\nu+1}{4\nu+1}\left((2\nu\sigma)^{(4\nu+1)/(2-2\nu)}-1\right)\right]$$

for the surface area of the maximum midsection.

Thus we find the head drag coefficient, the frontal surface area, and the mass of the body as functions of time for any ν.

4. Calculated Results. We now examine the results of calculations using the formulas presented above. Figure 3 shows the change in the shape of an originally spherical body (curve 1) for various combustion laws with $A = (2.48\ \mathrm{cm/sec})/(1.013\cdot10^5\ \mathrm{Pa})^\nu$ (curves 2-4 for $\nu = 1.0$, 0.5, and 0.27). Curves 2-4 were constructed for $\sigma = 0.4$, i.e., for the moment in time when the model reaches the end of the track [1]. It can be seen that a change in ν has a strong effect on the change of the lateral surface of the body near the maximum midsection.

Now we examine the case where the body has the shape of a parabola at time $t = 0$:

$$x_0=s,\quad y_0(s)=2ps^{1/2},\quad q_0=ps^{-1/2}.$$

The solution to the equation for the body shape change has the form

$$x(s, t) = s + \tau \frac{p^{2\nu-1}(2\nu s + p^2)}{(s + p^2)^{\nu+1/2}}, \quad y(s, t) = 2ps^{1/2} + \tau \frac{p^{2\nu}s^{1/2}(2\nu - 1)}{(s + p^2)^{\nu+1/2}}.$$

The radius of curvature at any point of the body is

$$\frac{R}{R_0} = 1 + \tau \frac{(2\nu - 1)\, p^{2\nu-1}(1/2p^2 - \nu s)}{(p^2 + s)^{3/2+\nu}}.$$

Figure 4 shows the shape change of a body which originally is a parabola (curve 1):

$$y/R_0 = 0{,}5\sqrt{x/R_0}.$$

Curves 2-4 in Fig. 4 correspond to ν = 1.0, 0.5, and 0.27 and σ = 0.2. Analogous (dashed) curves were calculated for an initial parabola with the equation

$$y/R_0 = 2\sqrt{x/R_0}.$$

Comparison of the calculations of y(x, t) shows that the change in ν has a stronger effect on the shape change of a thinner body than a thicker one.

Figure 5 shows calculations of the change in the wave-resistance coefficient of an initially spherical body as a function of the dimensionless time σ. Curves 1-3 correspond to ν = 1.0, 0.5, and 0.27. It can be seen that the coefficient C_p depends strongly on ν in the combustion equation.

Thus, the calculations show that ν is an important parameter in the shape change of a burning body. The value of ν can be established for a given material by analyzing photographs of the burning body at various times or else from the deceleration law for the body from the change in the coefficient C_p.

LITERATURE CITED

1. N. N. Baulin, D. G. Kuvalkin, N. N. Pilyugin, et al., "Investigation of the radiation brightness of gases near a burning model moving at hypersonic velocity," Kosm. Issled., 25, No. 1 (1987).
2. N. A. Anfimov, "Combustion of graphite in an air flow at high temperatures," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk Mekh. Mashinostr., No. 5 (1964).
3. F. A. Williams, Theory of Combustion [Russian translation], Nauka, Moscow (1971).
4. Ya. B. Zel'dovich, O. I. Leipunskii, and V. B. Librovich, Theory of Nonstationary Combustion of a Powder [in Russian], Nauka, Moscow (1975).
5. V. K. Bulgakov and A. M. Lipanov, "Interaction of turbulence and a chemical reaction in the theory of erosion combustion of solids," Khim. Fiz., 5, No. 4 (1986).
6. N. N. Pilyugin and R. F. Talipov, "Asymptotic theory of hypersonic flow around burning models and the determination of experimental constants," Jet and Discontinuous Flows [in Russian], Izdatel'stvo MGU (Moscow State University Publishing House), Moscow (1989).
7. N. N. Pilyugin and R. F. Talipov, "Asymptotic solution of the Euler equations in a shock layer during continuous flow around a blunt body and evolution of gas from its surface," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1989).
8. É. Z. Apshtein, N. N. Pilyugin, and G. A. Tirskii, "Mass erosion and shape change of a three-dimensional body moving along a trajectory in the Earth's atmosphere," Kosm. Issled., 17, No. 2 (1979).
9. L. E. Elsgold, Differential Equations and Variational Calculus [Russian translation], Nauka, Moscow (1965).